Technical Comments

Comment on "Does the Center of Flexure Depend on Poisson's Ratio?"

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DOES the center of flexure depend on Poisson's ratio? In his recent Note Leko[1] concludes that it does not, but a more accurate conclusion is that it all depends on how the center of flexure is defined. If the definition adopted by such authorities as Goodier,[2] Timoshenko,[3] and Sokolnikoff[4] is used, then the center of flexure definitely does depend on Poisson's ratio. This is evident from example calculations in the well-known treatises of Timoshenko[3] and Sokolnikoff.[4] On the other hand if Trefftz's[5] definition is used the center of flexure is independent of Poisson's ratio, as Trefftz pointed out in 1935. It is important to recognize that the Note under discussion is based on Leko's own definition of center of flexure, which is radically different from previous proposals.

Leko fixes the position of the center of flexure by introducing the requirement that "the resultant moment due to horizontal shear stresses is zero." This requirement differs sharply from traditional thinking in the classic problem of the center of flexure of a channel section. It is probably futile to argue about definitions, and interested readers will have to decide for themselves which of the various definitions of center of flexure is most natural or useful.

References

[1] Leko, T., "Does the Center of Flexure Depend on Poisson's Ratio?" *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1187–1188.

[2] Goodier, J. N., "A Theorem on the Shearing Stress in Beams with Applications to Multicellular Sections," *Journal of the Aeronautical Sciences*, Vol. 11, No. 3, July 1944, pp. 272–280.

[3] Timoshenko, S. P. and Goodier, J. N., *Theory of Elasticity*, 2nd ed., McGraw-Hill, New York, 1951.

[4] Sokolnikoff, I. S., *Mathematical Theory of Elasticity*, 2nd ed., McGraw-Hill, New York, 1956, pp. 204–209 and p. 239.

[5] Trefftz, E., "Über den Schubmittelpunkt in einem durch ein Einzellast gebogenen Balken," *Zeitschrift fuer Angewandte Math. und Mech.*, Vol. 15, No. 4, 1935, pp. 220–225.

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Reply by Author to G. R. Cowper

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WESTERGAARD wrote on the subject of the center of shear, "One might surmise that all the simple laws in mechanics of materials that deal with the bending of beams had been discovered before the end of the nineteenth century. Yet one matter had been overlooked, and the principle involved was not discovered and clarified until the 1920's which is strangely late."

However, as we know now, the precise determination of the center of flexure has not been resolved up to this date and there are a great number of recent publications that identify the location of the center of flexure with many different points of the cross section of the beam, so that further investigation is still desirable.

Reference

[1] Westergaard, H. M. *Theory of Elasticity*, Dover, New York, 1964, p. 22.

Received December 1, 1969. The author would like to thank G. R. Cowper for bringing up the question of definition of the center of flexure.

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Comment on "Vortex-Shedding from Circular Cylinders in Sheared Flow"

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Nomenclature

- f = shedding frequency from one side of the cylinder
- D = cylinder diameter
- U = local mean velocity
- U_c = center-line velocity
- y = ordinate measured from the center-line
- H = height of the test section
- λ = shear parameter = $(U - U_c)/(y/H)$
- S = Strouhal number = fD/U
- R = Reynolds number = UD/ν
- ν = kinematic viscosity

CHEN and Mangione[1] recently presented a correlation of local Strouhal number with local Reynolds number for sheared flows which agreed within ±10% with Roshko's correlation, viz.

$$S = 0.212 - (4.5/R)$$

Because they obtained agreement with Roshko's[2] data in the absence of their shear-generating grid, they attributed the scatter in their data to the turbulence intensity (4%) generated by this grid.

In the course of our study of the influence of the free-stream turbulence parameters on local heat transfer from cylinders in cross-flow,[3] we studied the effect of freestream turbulence level on the Strouhal number of both circular and square cylinders. The study was conducted in the 11 in. × 11 in. test section of a low-speed wind tunnel. Smooth Plexiglass cylinders ranging in diameter from $\frac{1}{4}$ in. to 1 in. were used to obtain a Reynolds number range of 4000 to 10,000. In the test section, the oncoming velocity profile was found to be flat. The shedding frequency was measured by placing a hot-wire probe in the near wake (about 2 diameters downstream and 0.5 diameter off the axis) and auto-

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correlating the turbulence output signal from the anemometer. The auto-correlations were computed in real time and plotted on an $X - Y$ plotter using different time delay ranges. The shedding frequency is readily obtainable from the auto-correlograms. The accuracy of the correlator was 2%. Grids of different geometrical configurations located at various distances upstream from the cylinder were used to study the effect of the freestream turbulence level and of scale on the frequency of eddy-shedding. The turbulence level ranged from about 0.5% for the clear tunnel to about 12.5%. Over the entire range of Reynolds number and turbulence intensity, the Strouhal number remained virtually constant at 0.205 ± 0.005. Similar experiments with square cylinders at zero angle of attack with the freestream indicated no noticeable effect of the turbulence intensity on the Strouhal number. It may be noted that for square cylinders the position of separation is fixed, and not dependent on the Reynolds number or the turbulence level.

In view of the preceeding findings, we suggest that the freestream level is not a significant factor in causing the large scatter in the data reported in Ref. 1. The shear parameter λ seems to be of primary importance. Also, since the flow past cylinders exposed to sheared freestream is distinctly three-dimensional, there appears to exist no reason a priori why Roshko's results for almost two-dimensional flow should apply in this case.

References

[1] Chen, C. F. and Mangione, B. J., "Vortex Shedding from Circular Cylinders in Sheared Flow," *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1211–1212.

[2] Roshko, A., "On the Drag and Shedding Frequency of Bluff Cylinders," TN 3169, 1954, NACA.

[3] Mujumdar, A. S., "Effect of Turbulence Parameters on Heat Transfer from Cylinders," Ph.D. thesis, McGill Univ., Montreal, Canada.

Reply by Author to Mujumdar and Douglas

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IN OUR experiment, the shear parameter λ is defined in terms of the tunnel height H, i.e.,

$$\lambda = (H/U_c)(dU/dy)$$

when U_c is the centerline velocity. The value of λ was found to be about 0.57. The shear parameter λ' based on the diameter d of the cylinder is

$$\lambda' = (d/U_c)(dU/dy) = \lambda(d/H) \sim 0.003$$

for the cylinders used. This means that in a vertical distance of 10 diameters, the velocity change is about 3%. With this fact in mind, it may not be too surprising to find that Roshko's correlation for uniform flow applies equally well in our case if the local values of Strouhal and Reynolds numbers are used. In view of Mujumdar's work on the effect of turbulence on vortex shedding, we should have blamed the scatter on our measuring technique (which is definitely not as sophisticated as that of Mujumdar) rather than the turbulence level.

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Comment on "Does the Center of Flexure Depend on Poisson's Ratio?"

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THE author[1] defines "center of flexure" in a manner contrary to the usual definition, and the particular property (dependence on Poisson's ratio) which the note examines happens to be exactly reversed by his definition. A partial definition is given at the beginning of the third paragraph and, slightly paraphrased, reads, "The center of flexure is whereever the load is applied." See also Eq. (10). A meaningful definition ought to say that the center of flexure is where the load must be applied to produce some prescribed effect. The accepted definition prescribes that the beam should deflect without twisting.† The author, instead, adds to his definition the statement "The natural requirement of bending by transversal vertical force is that the resultant moment due to horizontal shear stresses is zero." This definition is repeated mathematically in Eq. (5).

The fact that this definition contradicts universal usage is most easily illustrated by referring to a "thin-walled open section." In the absence of twisting, the shear is approximately uniform across the thickness and parallel to the boundaries. In a typical case (Fig. 1) the shear in the flanges *does* produce a moment, forcing the resultant vertical force completely outside the section. Furthermore, torsional shears to cancel this moment [Eq. (9)] would, of course, produce pronounced twisting. The remaining development is based on Eq. (5) and therefore invalid, and leads to a conclusion that is known to be wrong.

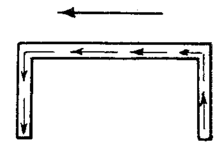

Fig. 1 Flexural center of open section.

References

[1] Leko, T., "Does the Center of Flexure Depend on Poisson's Ratio?," *AIAA Journal*, Vol. 7, No. 6, June 1969, pp. 1187–1188.

[2] Timoshenko, S. P. and Goodier, J. N., *Theory of Elasticity*, 3rd ed., McGraw-Hill, New York, 1970, pp. 371–374; also 2nd ed., 1951, pp. 333–336.

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† Since bending deforms the cross section a precise definition requires further refinement, as explained, for example, in Ref. 2, where, incidentally, Poisson's ratio does appear in the resulting expressions.

Reply by Author to D. B. Hall

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THE first sentence of D. B. Hall's comments states that the particular property of the center of flexure (independence of Poisson's ratio) is exactly reversed by the

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